Bohr Model of Hydrogen Atom

Postulates of Bohr Model

The electron revolves in discrete orbits around the nucleus without radiating any energy, contrary to what classical electromagnetism principle. In these orbits, the electron's acceleration does not result in radiation and energy loss.

These discrete orbits are called stationary orbits.

The electron cannot have any other orbit in between the discrete ones.

The angular momentum of electrons in these orbits is an integral multiple of the reduced Planck's constant.

$$mvr = n\hbar$$

Electron loses energy only when it jumps from one allowed energy level to another allowed energy level. It radiates energy in the form of electromagnetic radiation with a frequency ν determined by the energy difference of the levels according to the Planck relation:

$$\mathbf{E_2 - E_1 = h\nu}$$

Bohr assumed that during a jump a discrete or quantum of energy was radiated.

Bohr explained the quantization of the radiation emitted by the discreteness of the atomic energy levels.

Bohr did not believe in the existence of photons.

According to the Maxwell theory the frequency v of classical radiation is equal to the rotation frequency v of the electron in its orbit, with harmonics as integer multiples of this frequency.

This result is obtained from the Bohr model for jumps between energy levels E_n and E_{n-k} when k is much smaller than n. These jumps reproduce the frequency of the k^{th} harmonic of orbit n.

For sufficiently large values of n the two orbits involved in the emission process have nearly the same rotation frequency, thus giving credence to the classical orbital frequency. But for small n or large k, the classical interpretation fails to explain the radiation frequency.

This leads to the birth of the correspondence principle; quantum theory agrees with the classical theory only in the limit of large quantum numbers.

Bohr's model holds good for the hydrogen atom. Hydrogen atom is the simplest atom with one proton and one electron.

The model is also applicable to hydrogen like atoms or ions, e.g. He^{+}, Li^{2+}, Be^{3+} which have only one electron.

Drawbacks

Bohr model had several limitations and was replaced by the quantum mechanic s model.

The model stood strong in explaining spectra of lighter atoms similar to hydrogen. It could not explain the spectral lines for heavier atoms.

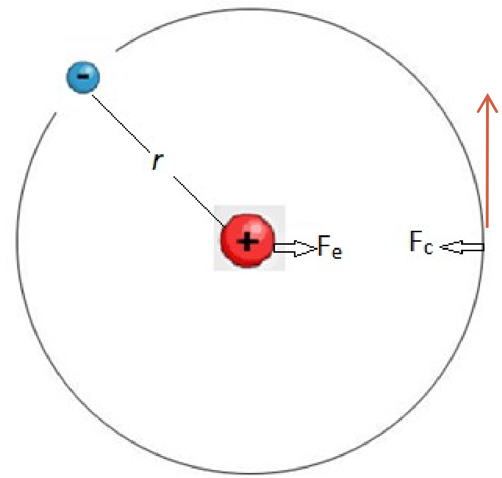
The model could not account for hyperfine structures like doublets and triplets.

As per the Bohr's model, the angular momentum of the electron in the ground state of a hydrogen atom is equal to the reduced Planck constant ($h/_{2\pi}$). The modern quantum theory says it is zero.

The model didn't stand to the advent of the dual nature (wave-particle duality) of the electron.

The model was in contradiction to the later development of the Heisenberg uncertainty principle that says that the position and momentum of a particle cannot be determined simultaneously. However, Bohr model defined the position (orbits) and momentum of the electron at the same time.

Assume a circular orbit for electrons for convenience



The centripetal force $F_c = \frac{mv^2}{r}$

This force holding the electrons in an orbit of radius r from the nucleus is provided by the centripetal force $F_e = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2}$

For stable orbits $F_c = F_e$ $\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2}$

The electron velocity v is related to the orbit radius r by the formula

$$v = \frac{e}{\sqrt{4\pi\varepsilon_o mr}}$$

Kinetic energy of electron in the orbit $T = \frac{1}{2} m v^2$

Potential energy of the system $V = -\frac{e^2}{4\pi\varepsilon_o r}$

The total energy E = T + V

$$E = \frac{1}{2} m v^2 - \frac{e^2}{4\pi\varepsilon_o r} = -\frac{e^2}{8\pi\varepsilon_o r}$$

The total energy of the atomic electron is negative. This is necessary for a bound orbit.

If the energy is zero or positive the electron is not bound to the nucleus. Experiments show that 13.6 ev energy is needed to remove the electron from the hydrogen atom orbit.

$$r = -\frac{e^2}{8\pi\varepsilon_o E} = 5.3 \times 10^{-11} m$$

Wave behaviour of electron in the Bohr orbit $\lambda = \dfrac{h}{mv}$

The electron speed v is given by $v = \dfrac{e}{\sqrt{4\pi\varepsilon_0 m r}} = 33 \times 10^{-11} m$

$$\lambda = \frac{h}{e}\sqrt{\frac{4\pi\varepsilon_0 r}{m}} = 33 \times 10^{-11} m$$

The circumference of the first Bohr orbit is $2\pi r = 33 \times 10^{-11} m$

The circumference is same as the wavelength of the electron. The orbit of the electron in a hydrogen atom corresponds to one complete electron wave joined on itself.

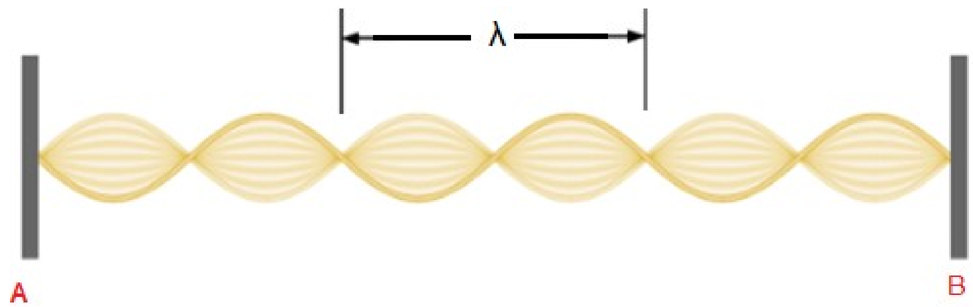
The fact that the electron orbit in a hydrogen atom is one electron wavelength in circumference provides the clue needed to construct a theory of the atom.

Using the concept of the electron matter wave, de Broglie provided a rationale for the quantization of the electron's angular momentum in the hydrogen atom, which was postulated in Bohr's quantum theory.

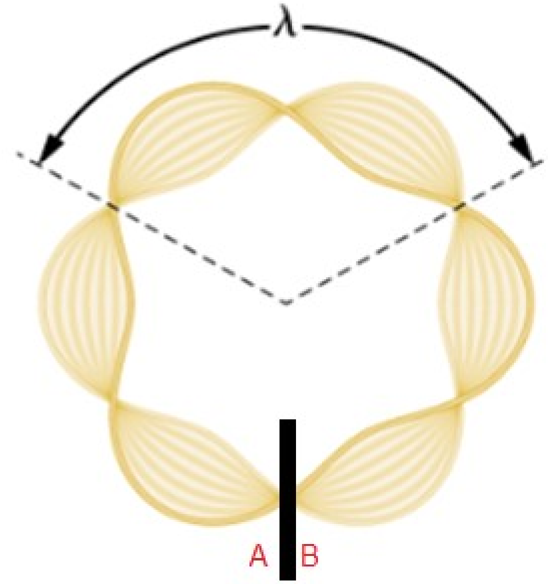
The physical explanation for the first Bohr quantization condition comes naturally when we assume that an electron in a hydrogen atom behaves not like a particle but like a wave.

Imagine a stretched string that is clamped at both ends and vibrates in one of its normal modes. If the length of the string is l, the wavelengths of these vibrations cannot be arbitrary but must be such that an integer k number of half-wavelengths fit exactly on the distance l between the ends.

This is the condition for a standing wave on a string.



Now suppose we bend its length into a circle and fasten its ends to each other.



This produces a circular string that vibrates in normal modes, satisfying the same standing-wave condition, but the number of half-wavelengths must now be an even number and the length *l* is now circumference of the circle.

This means that the radii in Bohr model are not arbitrary but must satisfy the following standing-wave condition:

$$2\pi r_n = 2n\frac{\lambda}{2}.$$

If an electron in the *n*th Bohr orbit moves as a wave, its wavelength should be equal to

$$\lambda = 2\pi r_n / n.$$

The electron wave of this wavelength corresponds to the electron's linear momentum,

$$p = h / \lambda = nh / (2\pi r_n) = n\hbar / r_n.$$

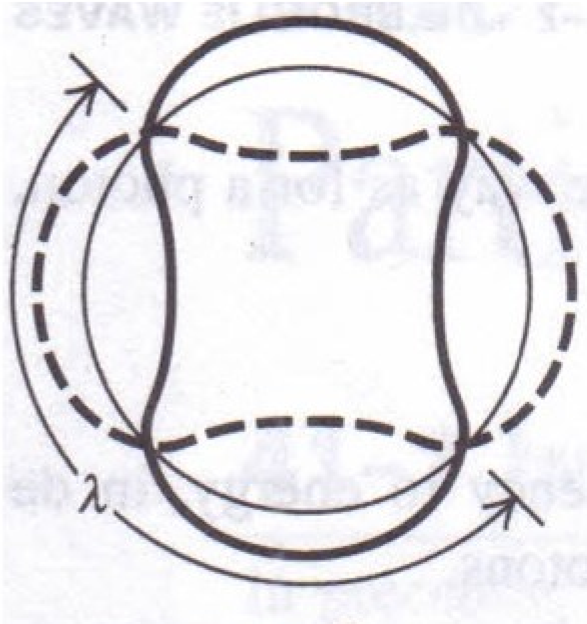
In a circular orbit, therefore, the electron's angular momentum must be

$$L_n = r_n p = r_n \frac{n\hbar}{r_n} = n\hbar.$$

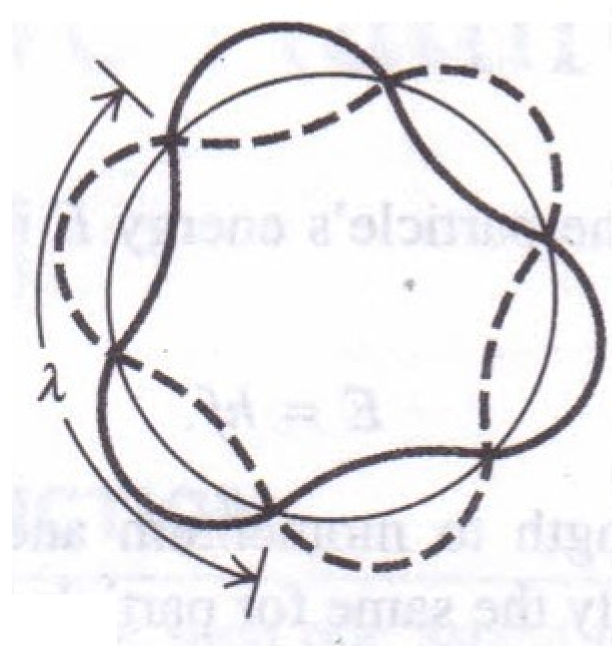
This equation is the first of Bohr's quantization conditions.

It provides physical explanation for Bohr's quantization condition.

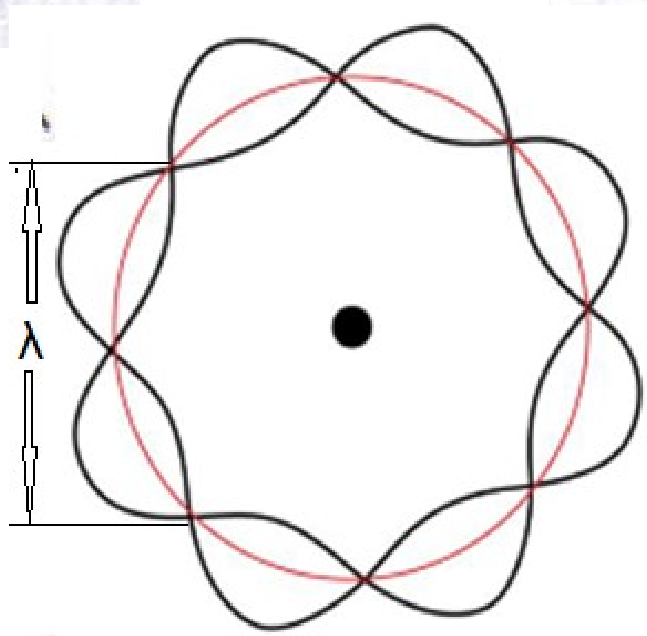
It is a convincing theoretical argument for the existence of matter waves.



$n = 2$



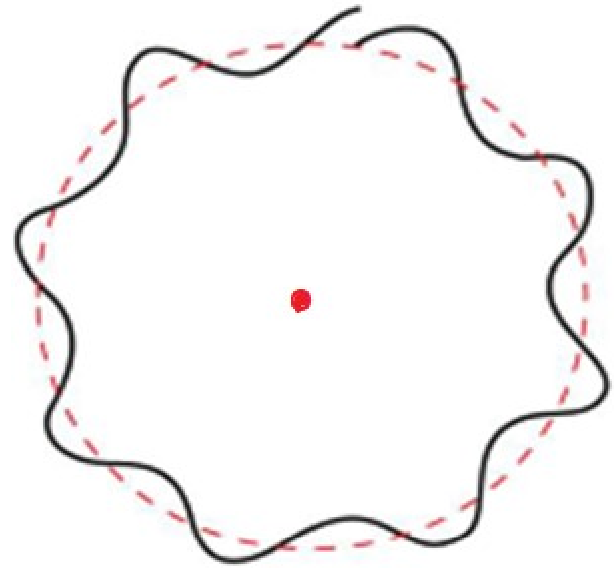
$n = 3$



n=4

Why are these only vibrations possible in a loop?

If a fractional number of wavelength is placed around the loop destructive interference will occur as the waves travel around the loop, and the vibrations will die out rapidly.



By considering the behavior of electron waves in the hydrogen as analogous to the vibrations of a wire loop we may postulate that an electron can circle a nucleus indefinitely without radiating energy provided that the orbit contains an integral number of de Broglie wavelengths.

This postulate combines both the particle and wave characters of an electron into a single statement, since the electron wavelength is computed from the orbital speed required to balance the electrostatic attraction of the nucleus.

While we can never observe these antithetical characters simultaneously, they are inseparable in nature.

It is a simple matter to express the condition that an electron orbit contains an integral number of de Broglie wavelengths. The circumference of the circular orbit of radius r is 2πr, so we may write the condition for stability as

$$2\pi r_n = n\lambda, \qquad n = 1, 2, 3, \dots\dots$$

So the stable electron radii are $$r_n = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2} \qquad n = 1, 2, 3, \dots.$$

Energy E_n is given by $$E_n = -\frac{e^2}{8\pi\varepsilon_o r}$$

Substituting for r_n $$E_n = -\frac{me^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{n^2}\right)$$ n=1, 2, 3,……..

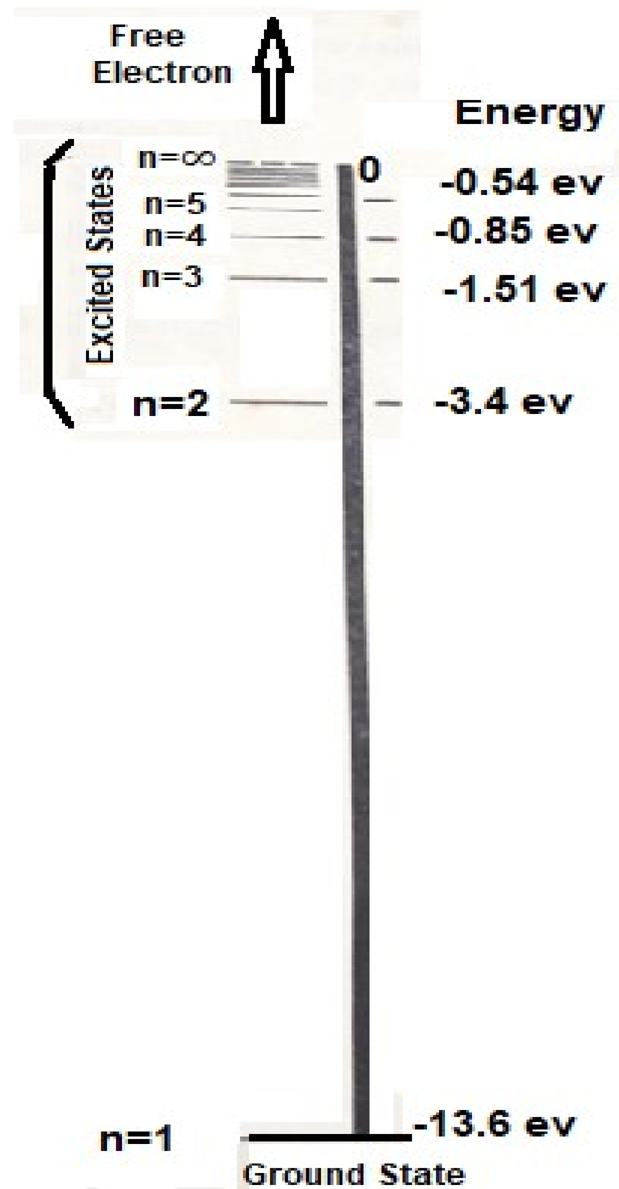
Energies specified by this equation are called the energy levels of hydrogen atom. The levels are all negative, signifying that the electron doesn't have enough energy to escape from the atom. The energy level E_1 is called the ground state while E_2, E_3, E_4 are called excited states.

As the quantum numbers increase the corresponding energy E_n increases and approaches 0 in the limit of $n \rightarrow \infty$, $E_\infty = 0$ and the electron is no longer bound to the nucleus to form the atom.

The ground state energy E_1 of the hydrogen atom is a convenient energy unit for use in various aspects of atomic and molecular physics. This energy is called the rydberg (ry) and its numerical value is

$$1\,ry = \frac{me^4}{8\varepsilon_o^2 h^2}$$

$$= 2.17 \times 10^{-18}\,joule = 13.6\,ev$$



When an electron in an excited state drops to a lower state the lost energy is emitted as a single photon of light. According to the model an electron cannot reside in an atom except in certain specified orbits. The quantum number of the initial state (higher energy level) is n_i and that of final state (lower enrgy state) is n_f then

Initial energy – final energy = photon energy $\mathbf{E_i - E_f = h\nu}$

$$\text{Initial energy} = E_i = -\frac{me^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{n_i^2}\right) \qquad \text{Final Energy} = E_f = -\frac{me^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{n_f^2}\right)$$

Energy difference between these two states

$$E_i - E_f = \frac{me^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{-n_i^2}\right) - \left(-\frac{1}{n_f^2}\right)$$

$$E_i - E_f = \frac{me^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The frequency of the photon released in emission is

$$\nu = \frac{E_i - E_f}{h} = \frac{me^4}{8\varepsilon_o^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

In terms of photon wavelength $\lambda = \frac{c}{\nu}$

We have $$\frac{1}{\lambda} = \frac{\nu}{c} = \frac{me^4}{8\varepsilon_o^2 c h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

This expression states that the radiation emitted by excited hydrogen atoms should contain certain wavelengths only.

These wavelengths fall into definite sequences that depend upon the quantum numbers n_f of the final state of the electron.

The calculated formulae for the first five series are

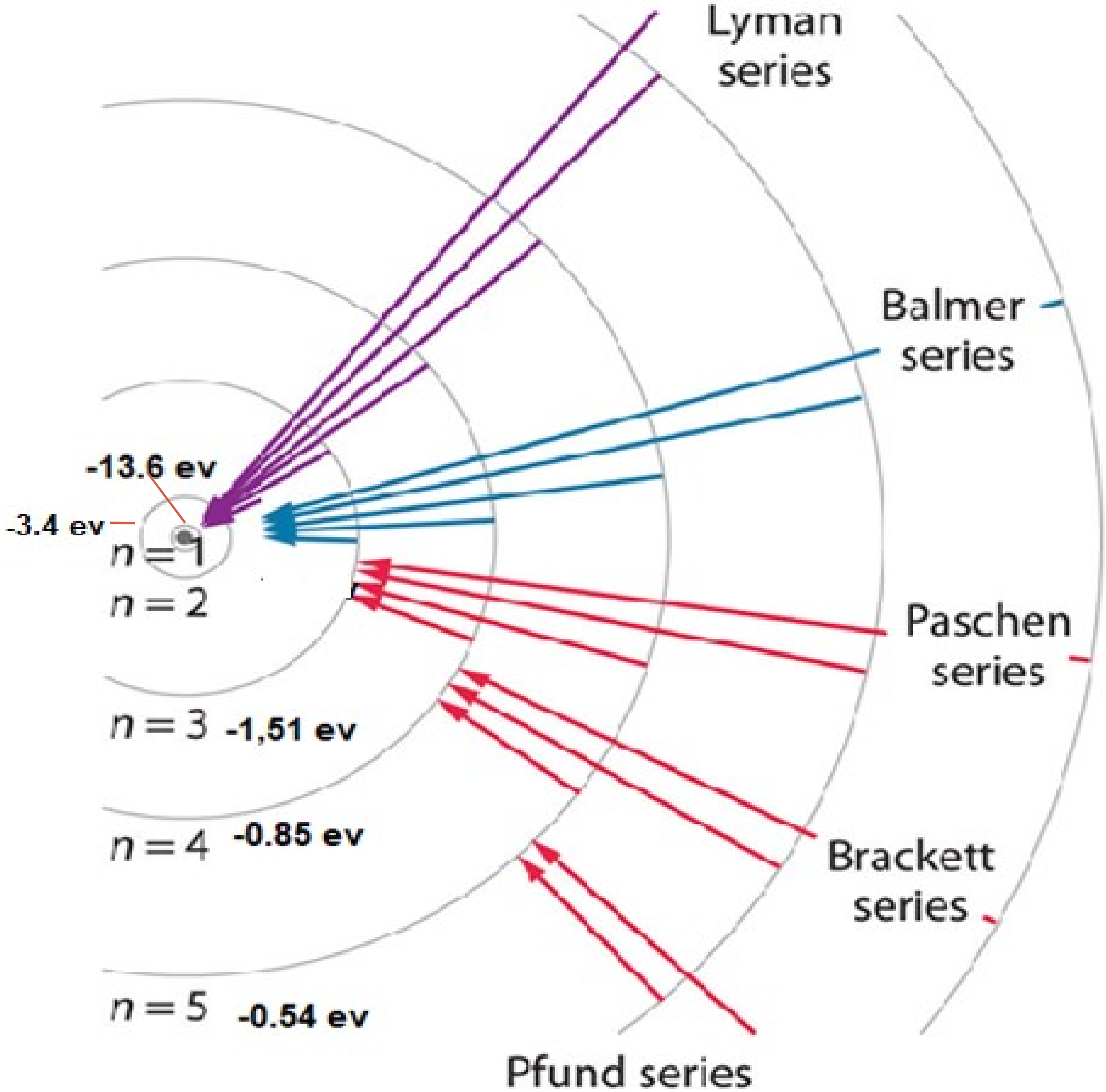
$$n_f = 1, \quad \frac{1}{\lambda} = \frac{me^4}{8\varepsilon_o^2 ch^3}\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \quad n = 2, 3, 4, \ldots \ldots \text{Lyman Series}$$

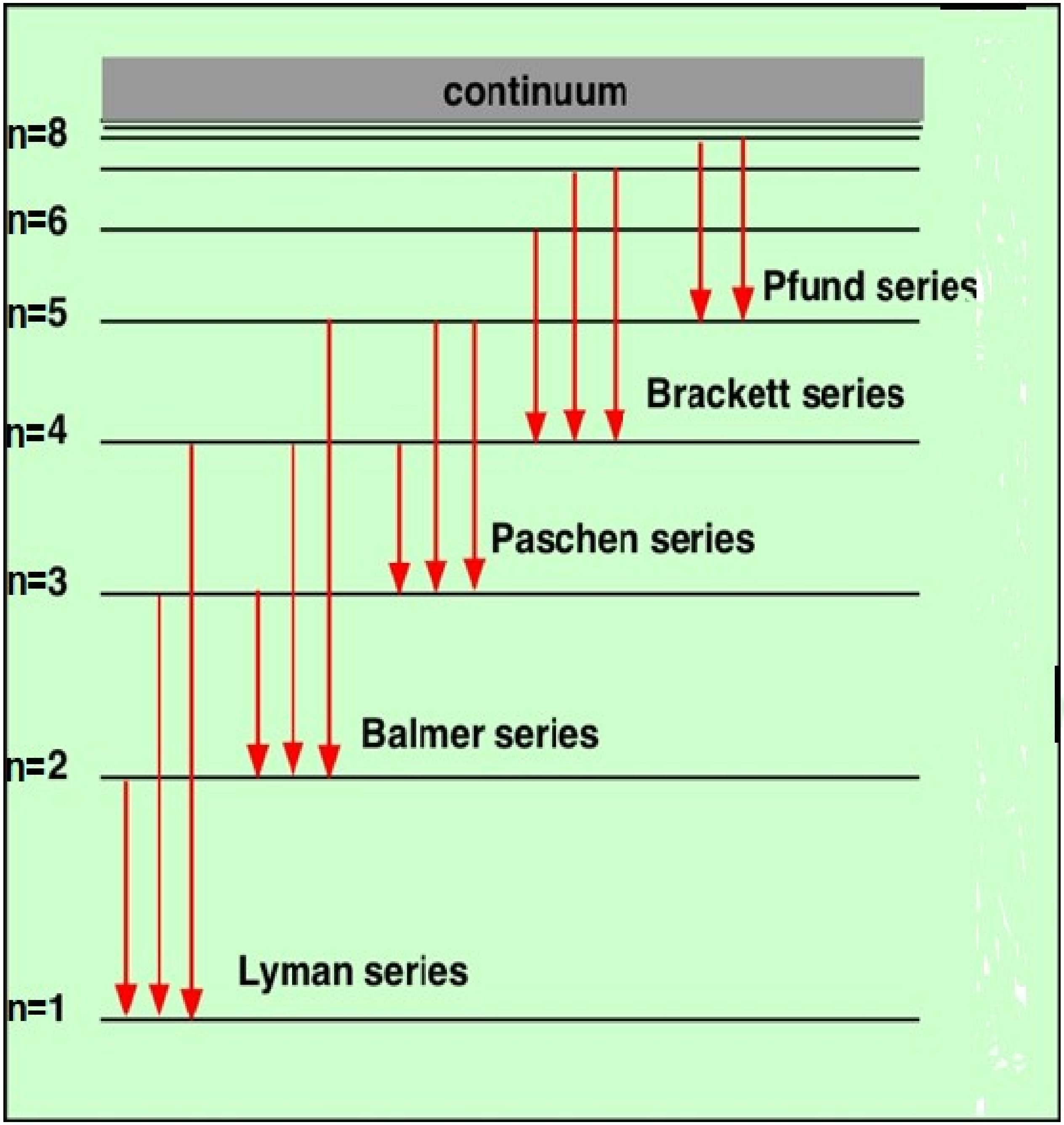
$$n_f = 2, \quad \frac{1}{\lambda} = \frac{me^4}{8\varepsilon_o^2 ch^3}\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n = 3, 4, 5, \ldots \ldots . Balmer\ Series$$

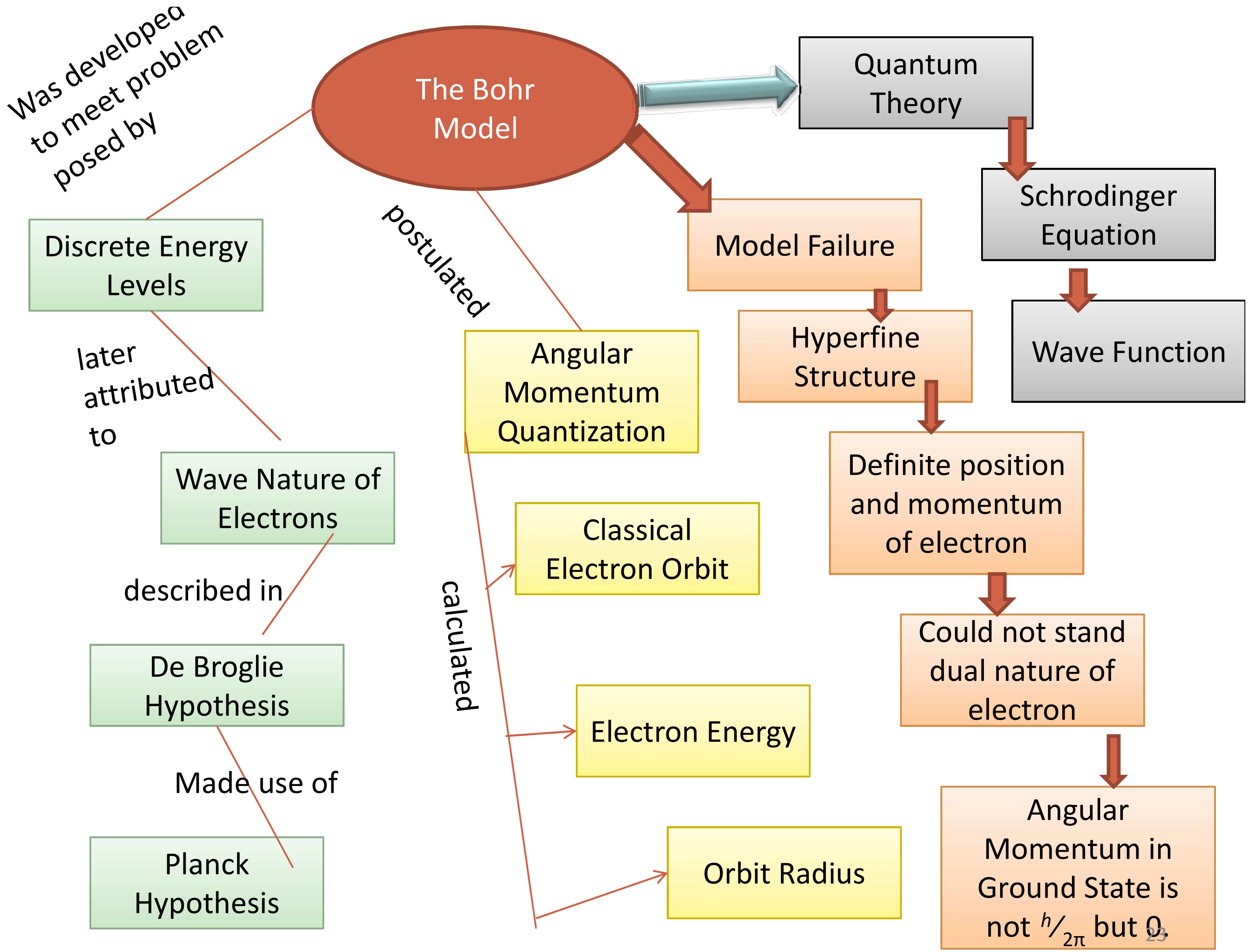
$$n_f = 3, \quad \frac{1}{\lambda} = \frac{me^4}{8\varepsilon_o^2 ch^3}\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \quad n = 4, 5, 6 \ldots \ldots . Paschen\ Series$$

$$n_f = 4, \quad \frac{1}{\lambda} = \frac{me^4}{8\varepsilon_o^2 ch^3}\left(\frac{1}{4^2} - \frac{1}{n^2}\right) \quad n = 5, 6, 7 \ldots \ldots . Bracket\ Series$$

$$n_f = 5, \quad \frac{1}{\lambda} = \frac{me^4}{8\varepsilon_o^2 ch^3}\left(\frac{1}{5^2} - \frac{1}{n^2}\right) \quad n = 6, 7, 8 \ldots \ldots . Pfund\ Series$$


Lyman series
Balmer series
Paschen series
Brackett series
Pfund series
-13.6 ev
-3.4 ev
n = 1
n = 2
n = 3 -1,51 ev
n = 4 -0.85 ev
n = 5 -0.54 ev


continuum
n=8
n=6
n=5
n=4
n=3
n=2
n=1
Pfund series
Brackett series
Paschen series
Balmer series
Lyman series


The Bohr Model
Was developed to meet problem posed by
Discrete Energy Levels
later attributed to
Wave Nature of Electrons
described in
De Broglie Hypothesis
Made use of
Planck Hypothesis
postulated
Angular Momentum Quantization
calculated
Classical Electron Orbit
Electron Energy
Orbit Radius
Quantum Theory
Schrodinger Equation
Wave Function
Model Failure
Hyperfine Structure
Definite position and momentum of electron
Could not stand dual nature of electron
Angular Momentum in Ground State is not $h/2\pi$ but 0.

Erwin Schrödinger
Austrian physicist

Louis de Broglie
French physicist

De Broglie won the Nobel Prize for Physics in 1929, after the wave-like behaviour of matter was first experimentally demonstrated in 1927.

Originally, de Broglie thought that real wave having a direct physical interpretation was associated with particles.

The wave aspect of matter was formalized by a wave function defined by the Schrödinger equation.

Wave function is a pure mathematical entity. It is a complex function. It has a probabilistic interpretation, without the support of real physical elements. This wave function gives an appearance of wave behavior to matter, without making real physical waves appear.

Prof. Narendra Kumar Pandey
Department of Physics
University of Lucknow
Lucknow-226007
Email: profnarendrapandey137@gmail.com